Statistical Natural Language Processing
Statistical Parsing

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This lecture is about statistical constituency and dependency parsing of natural languages.
Why do we need syntactic parsing?

- Syntactic analysis is an intermediate step in (semantic) interpretation of sentences

```
S  
 NP  
  John  
 VP  
  V  
  saw  
 NP  
  Mary 
```

As result, it is useful for applications like question answering, information extraction, ...

- (Statistical) parsers can also be used as language models for applications like speech recognition and machine translation

- It can be used for grammar checking, and can be a useful tool for linguistic research
Ingredients of a parser

- A grammar
- An algorithm for parsing
- A method for ambiguity resolution
Dependency vs. constituency

- Constituency grammars are based on units formed by a group of lexical items (constituents or phrases)
- Dependency grammars model binary head-dependent relations between words
- Most of the theory of parsing is developed with constituency grammars
- Dependency grammars has recently become popular in CL

\[
\begin{array}{c}
S \\
\downarrow \\
NP \quad VP \\
\downarrow \\
V \quad NP \\
\downarrow \\
John \quad saw \quad Mary
\end{array}
\]
Constituency grammars

- Constituency grammars are probably the most studied grammars both in linguistics, and computer science.
- The main idea is that groups of words form natural groups, or ‘constituents’, like *noun phrases* or *word phrases*.
- *Phrase structure grammars* or *context-free grammars* are often used as synonyms.

Note: many grammar formalisms posit a particular form of constituency grammars, we will not focus on a particular grammar formalism here.
Formal definition

A phrase structure grammar is a tuple \((\Sigma, N, S, R)\)

- \(\Sigma\) is a set of terminal symbols
- \(N\) is a set of non-terminal symbols
- \(S \in N\) is a distinguished start symbol
- \(R\) is a set of 'rewrite' rules of the form \(\alpha A \beta \rightarrow \gamma\) for \(A \in N\), \(\alpha, \beta, \gamma \in \Sigma \cup N\)

The grammar accepts a sentence if it can be derived from \(S\) with the rewrite rules \(R\).
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The grammar accepts a sentence if it can be derived from \(S\) with the rewrite rules \(R\)

\[
\begin{align*}
S & \rightarrow NP \ VP \\
NP & \rightarrow John \mid Mary \\
VP & \rightarrow V NP \\
V & \rightarrow saw
\end{align*}
\]
Formal definition

A phrase structure grammar is a tuple \((\Sigma, N, S, R)\)

- \(\Sigma\) is a set of terminal symbols
- \(N\) is a set of non-terminal symbols
- \(S \in N\) is a distinguished start symbol
- \(R\) is a set of ‘rewrite’ rules of the form \(\alpha A \beta \rightarrow \gamma\) for \(A \in N\) and \(\alpha, \beta, \gamma \in \Sigma \cup N\)

- The grammar accepts a sentence if it can be derived from \(S\) with the rewrite rules \(R\)

\[
S \rightarrow NP \ VP \\
NP \rightarrow John \mid Mary \\
VP \rightarrow V \ NP \\
NP \rightarrow John \mid Mary \\
V \rightarrow saw
\]
Example derivation

The example grammar:

\[
\begin{align*}
S & \rightarrow \ NP \ VP \\
NP & \rightarrow \ John \ | \ Mary \\
VP & \rightarrow \ V \ NP \\
V & \rightarrow \ saw
\end{align*}
\]

- Phrase structure grammars derive a sentence with successive application of rewrite rules.

\[
\begin{align*}
S & \Rightarrow NP \ VP \Rightarrow John \ VP \Rightarrow John \ V \ NP \Rightarrow John \ saw \ NP \Rightarrow John \ saw \ Mary \\
\text{or, } S & \Rightarrow John \ saw \ Mary
\end{align*}
\]

- The intermediate forms that contain non-terminals are called *sentential forms*
Constituency grammars and parsing

- Context-free grammars are parseable in $O(n^3)$ time complexity using dynamic programming algorithms.
- Mildly context-sensitive grammars can also be parsed in polynomial time ($O(n^6)$).
- Polynomial time algorithms are not always fast enough in practice.
  - We often use approximate solutions with greedy search algorithms.
Where do grammars come from

- Grammars for (statistical) parsing can be either
  - hand crafted (many years of expert effort)
  - extracted from *treebanks* (which also require lots of effort)
  - ‘induced’ from raw data (interesting, but not as successful)
- Current practice relies mostly on treebanks
- Hybrid approaches also exist
- Grammar induction is not common (for practical models) but exploiting unlabeled data is also a common trend
Context free grammars

recap

- Context free grammars are sufficient for expressing most phenomena in natural language syntax
- Most of the parsing theory (and practice) is build on parsing CF languages
- The context-free rules have the form

  \[ A \rightarrow \alpha \]

  where \( A \) is a single non-terminal symbol and \( \alpha \) is a (possibly empty) sequence of terminal or non-terminal symbols
An example context-free grammar

\[
\begin{align*}
S & \rightarrow NP \ VP \\
S & \rightarrow Aux \ NP \ VP \\
NP & \rightarrow Det \ N \\
NP & \rightarrow Prn \\
NP & \rightarrow NP \ PP \\
VP & \rightarrow V \ NP \\
VP & \rightarrow V \\
VP & \rightarrow VP \ PP \\
PP & \rightarrow Prp \ NP \\
N & \rightarrow duck \\
N & \rightarrow park \\
N & \rightarrow parks \\
V & \rightarrow duck \\
V & \rightarrow ducks \\
V & \rightarrow saw \\
Prn & \rightarrow she \mid her \\
Prp & \rightarrow in \mid with \\
Det & \rightarrow a \mid the
\end{align*}
\]

Derivation of sentence ‘she saw a duck’

\[
S \Rightarrow NP \ VP \\
NP \Rightarrow Prn \\
Prn \Rightarrow she \\
VP \Rightarrow V \ NP \\
V \Rightarrow saw \\
NP \Rightarrow Det \ N \\
Det \Rightarrow a \\
N \Rightarrow duck
\]

\[
\text{she} \quad \text{saw} \quad \text{a} \quad \text{duck}
\]
Representations of a context-free parse tree

A parse tree:

```
S
 /\   /
|  |  |
NP VP
```

A history of derivations:

- $S \Rightarrow \text{NP } \text{VP}$
- $\text{NP } \Rightarrow \text{Prn}$
- $\text{Prn } \Rightarrow \text{I}$
- $\text{VP } \Rightarrow \text{V } \text{NP}$
- $\text{V } \Rightarrow \text{saw}$
- $\text{NP } \Rightarrow \text{Prn}_p \text{ N}$
- $\text{Prn}_p \Rightarrow \text{her}$
- $\text{N } \Rightarrow \text{duck}$

A sequence with (labeled) brackets

```
[S[NP[Prn I]][VP[V saw][NP[Prn_p her][N duck]]]]
```
Parsing as search

- Parsing can be seen as search constrained by the grammar and the input.
- Top down: start from S, find the derivations that lead to the sentence.
- Bottom up: start from the sentence, find series of derivations (in reverse) that leads to S.
- Search can be depth first or breadth first for both cases.
Problems with search procedures

- Top-down search considers productions incompatible with the input, and cannot handle left recursion
- Bottom-up search considers non-terminals that would never lead to $S$
- Repeated work because of backtracking
  $\rightarrow$ The result is exponential time complexity in the length of the sentence

Some of these problems can be solved using \textit{dynamic programming}.
CKY algorithm

- The CKY (Cocke–Younger–Kasami), or CYK, parsing algorithm is a dynamic programming algorithm
- It processes the input *bottom up*, and saves the intermediate results on a *chart*
- Time complexity for *recognition* is $O(n^3)$ (with a space complexity of $O(n^2)$)
- It requires the CFG to be in *Chomsky normal form* (CNF)
Chomsky normal form (CNF)

- A CFG is in CNF, if the rewrite rules are in one of the following forms
  - \( A \to B \ C \)
  - \( A \to a \)
  
where \( A, B, C \) are non-terminals and \( a \) is a terminal

- Any CFG can be converted to CNF
- Resulting grammar is \textit{weakly equivalent} to the original grammar:
  - it generates/accepts the same language
  - but the derivations are different
Converting to CNF: example

- For rules with > 2 RHS symbols
  \[ S \rightarrow \text{Aux NP VP} \quad \Rightarrow \quad S \rightarrow \text{Aux } X \]
  \[ X \rightarrow \text{NP VP} \]

- For rules with < 2 RHS symbols
  \[ \text{NP } \rightarrow \text{Prn} \quad \Rightarrow \quad \text{NP } \rightarrow \text{she | her} \]
CKY demonstration

recognition example

she  saw  her  duck
CKY demonstration

recognition example

```
s → NP VP
VP → V NP
NP → Prn N

S → NP VP
VP → V S
VP → V, VP
NP → Prn, NP
NP → N, V, VP
```

she saw her duck

Prn, NP V, VP Prn, NP N, V, VP
CKY demonstration

recognition example

```
S  →  NP VP

S  →  NP VP
V, VP
Prn, NP
Prn, NP
N, V, VP
she
saw
her
duck
```
CKY demonstration

recognition example

\[
\begin{align*}
VP & \rightarrow V NP \\
S & \rightarrow NP VP \\
\text{she} & \text{VP} \\
\text{saw} & \text{NP} \\
\text{her} & \text{V, VP} \\
\text{duck} & \text{Prn, NP}
\end{align*}
\]
CKY demonstration

recognition example

NP  →  Prn N
S  →  NP VP

S  →  NP VP
VP  →  V NP
NP  →  Prn N
S  →  NP VP
VP  →  V S
S  →  NP VP

she  saw  her  duck

0  1  2  3  4
Prn, NP  V, VP  Prn, NP  N, V, VP
CKY demonstration

recognition example

she saw her duck

S

VP

NP, S

Prn, NP

V, VP

Prn, NP

N, V, VP

0

1

2

3

4
CKY demonstration

recognition example

S → NP VP

S

NP, S

V, VP

Prn, NP

she

saw

her

duck

Prn, NP

NP, S

VP

S

0 1 2 3 4
CKY demonstration

recognition example

she saw her duck

Prn, NP V, VP Prn, NP N, V, VP

S VP NP, S

S → NP VP

VP → V NP

NP → Prn N

S → NP VP

S → NP VP

VP → V S
CKY demonstration

recognition example

she saw her duck

Prn, NP  V, VP  Prn, NP  N, V, VP

S  VP  NP, S

0  1  2  3  4
CKY demonstration

recognition example

```
Prn, NP  V, VP  Prn, NP  N, V, VP

S       VP     S       NP, S

VP → V NP
VP → V S
```

she saw her duck
CKY demonstration

recognition example

S

VP

NP, S

Prn, NP

V, VP

Prn, NP

N, V, VP

she

saw

her

duck
CKY demonstration

recognition example

she saw her duck

Prn, NP V, VP Prn, NP N, V, VP

S VP NP, S

S VP NP

S VP S

S → NP VP

VP → V NP

NP → Prn N

N, V, VP

Prn, NP V, VP
CKY demonstration

recognition example

she saw her duck

S → NP VP

Prn, NP V, VP Prn, NP N, V, VP

S V, VP S VP S NP, S

S → NP VP S → NP VP VP → V NP VP → V S NP → Prn N
CKY demonstration

recognition example

she saw her duck

Prn, NP V, VP Prn, NP N, V, VP

S VP NP, S

S VP S

S → NP VP

VP → V NP

S → NP VP

VP → V S
CKY demonstration

recognition example
CKY demonstration: the chart

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>S</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>V, VP</td>
<td>VP</td>
<td>VP</td>
<td></td>
</tr>
<tr>
<td>Prn</td>
<td>NP, S</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0  she  1  saw  2  her  3  duck  4

Chart is a 2-dimensional array, hence $O(n^2)$ space complexity.
Parsing requires back pointers

she saw her duck

Prn, NP V, VP Prn, NP N, V, VP

S, S V, VP NP, S

S S VP, VP

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CKY summary

+ CKY avoids re-computing the analyses by storing the earlier analyses (of sub-spans) in a table
– It still computes lower level constituents that are not allowed by the grammar
– CKY requires the grammar to be in CNF
  • CKY has $O(n^3)$ recognition complexity
  • For parsing we need to keep track of backlinks
  • CKY can efficiently store all possible parses in a chart
  • Enumerating all possible parses have exponential complexity (worst case)
Earley algorithm

- Earley algorithm is a top down parsing algorithm
- It allows arbitrary CFGs
- Keeps record of constituents that are
  predicted using the grammar (top-down)
  in-progress with partial evidence
  completed based on input seen so far
  at every position in the input string
- Time complexity is $O(n^3)$
Summary: context-free parsing algorithms

- Naive search for parsing is intractable
- Dynamic programming algorithms allow polynomial time recognition
- Parsing may still be exponential in the worse case
- Ambiguity: CKY or Earley parse tables can represent ambiguity, but cannot say anything about which parse is the best
Pretty little girl’s school (again)

The task: choosing the most plausible parse

We saw the man with a hat

We saw the man with a hat
The task: choosing the most plausible parse

We saw the man with a hat

\[
S \\
NP \quad VP \\
\quad V \quad NP \\
\quad \quad NP \quad PP \\
\quad \quad \quad D \quad N \\
\quad \quad \quad P \quad NP \\
\quad \quad \quad \quad D \quad N
\]

We saw the man with a hat

\[
S \\
NP \quad VP \\
\quad VP \quad PP \\
\quad V \quad NP \\
\quad \quad P \quad NP \\
\quad \quad \quad D \quad N \\
\quad \quad \quad D \quad N
\]
Statistical parsing

- Find the most plausible parse of an input string given all possible parses
- We need a scoring function, for each parse, given the input
- We typically use probabilities for scoring, task becomes finding the parse (or tree), $t$, given the input string $w$

$$t_{\text{best}} = \arg \max_{t} P(t \mid w)$$

- Note that some ambiguities need a larger context than the sentence to be resolved correctly
Probabilistic context free grammars (PCFG)

A probabilistic context free grammar is specified by,

- $\Sigma$ is a set of terminal symbols
- $N$ is a set of non-terminal symbols
- $S \in N$ is a distinguished start symbol
- $R$ is a set of rules of the form

$$A \rightarrow \alpha \quad [p]$$

where $A$ is a non-terminal, $\alpha$ is string of terminals and non-terminals, and $p$ is the probability associated with the rule

- The grammar accepts a sentence if it can be derived from $S$ with rules $R_1 \ldots R_k$
- The probability of a parse $t$ of input string $w$, $P(t \mid w)$, corresponding to the derivation $R_1 \ldots R_k$ is

$$P(t \mid w) = \prod_{i=1}^{k} p(R_i)$$

where $p(R_i)$ is the probability of the rule $R_i$
PCFG example (1)

We saw the man with a hat

S → NP VP 1.0
NP → D N 0.7
NP → NP PP 0.2
NP → We 0.1
VP → V NP 0.9
VP → VP PP 0.1
PP → P NP 1.0
N → hat 0.2
N → man 0.8
V → saw 1.0
P → with 1.0
D → a 0.6
D → the 0.4
PCFG example (1)

S → NP VP 1.0
NP → D N 0.7
NP → NP PP 0.2
NP → We 0.1
VP → V NP 0.9
VP → VP PP 0.1
PP → P NP 1.0
N → hat 0.2
N → man 0.8
V → saw 1.0
P → with 1.0
D → a 0.6
D → the 0.4

P(t) = 1.0 × 0.1 × 0.9 × 1.0 × 0.2 × 0.7 × 0.4 × 0.8 × 1.0 × 1.0 × 0.7 × 0.6 × 0.2
= 0.000263424
PCFG example (2)

S → NP VP 1.0
NP → D N 0.7
NP → NP PP 0.2
NP → We 0.1
VP → V NP 0.9
VP → VP PP 0.1
PP → P NP 1.0
N → hat 0.2
N → man 0.8
V → saw 1.0
P → with 1.0
D → a 0.6
D → the 0.4

We saw the man with a hat
PCFG example (2)

S → NP VP 1.0
NP → D N 0.7
NP → NP PP 0.2
NP → We 0.1
VP → V NP 0.9
VP → VP PP 0.1
PP → P NP 1.0
N → hat 0.2
N → man 0.8
V → saw 1.0
P → with 1.0
D → a 0.6
D → the 0.4

P(t) = 1.0 × 0.1 × 0.1 × 0.9 × 1.0 × 0.7 × 0.4 × 0.8 × 1.0 × 1.0 × 0.7 × 0.6 × 0.2
= 0.0001693440
Where do the rule probabilities come from?

- Supervised: estimate from a treebank, e.g., using maximum likelihood estimation
- Unsupervised: expectation-maximization (EM)
PCFGs - an interim summary

- PCFGs assign probabilities to parses based on CFG rules used during the parse.
- PCFGs assume that the rules are independent.
- PCFGs are generative models, they assign probabilities to $P(t, w)$, we can calculate the probability of a sentence by

$$P(w) = \sum_t P(t, w) = \sum_t P(t)$$
PCFG chart parsing

- Both CKY and Earley algorithms can be adapted to PCFG parsing
- CKY matches PCFG parsing quite well
  - to get the best parse, store the constituent with the highest probability in every cell of the chart
  - to get n-best best parse (beam search), store the n-best constituents in every cell in the chart
CKY for PCFG parsing

I saw her duck

S → NP VP
VP → V NP
NP → Prn N
S → NP VP
VP → V S

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CKY for PCFG parsing

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Prn, NP</td>
<td>V, VP</td>
<td>Prn, NP</td>
<td>N, V, VP</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>saw</td>
<td>her</td>
<td>duck</td>
<td></td>
</tr>
</tbody>
</table>

\[
P(\text{Prn}_{01}) = P(\text{Prn} \rightarrow I) \quad P(\text{NP}_{01}) = P(\text{NP} \rightarrow I)
\]
\[
P(\text{V}_{12}) = P(\text{V} \rightarrow \text{saw}) \quad P(\text{VP}_{12}) = P(\text{VP} \rightarrow \text{saw})
\]

\[
\ldots
\]
**CKY for PCFG parsing**

\[
P(S_02 \Rightarrow NP_{01} VP_{12}) = P(NP_{01}) P(VP_{12}) P(S \rightarrow NP \ VP)
\]
CKY for PCFG parsing

\[
P(VP_{13} \Rightarrow V_{12}NP_{23}) = P(V_{12})P(NP_{23})P(VP \rightarrow V NP)
\]
CKY for PCFG parsing

\[ P(NP_{24} \Rightarrow \text{Prn}_{23}N_{34}) = P(\text{Prn}_{23})P(N_{34})P(\text{Prn} \rightarrow \text{Prn } N) \]

\[ > \]

\[ P(S_{24} \Rightarrow NP_{23}VP_{34}) = P(NP_{23})P(VP_{34})P(S \rightarrow NP \ VP) \]
CKY for PCFG parsing

I saw her duck

S → NP VP
VP → V NP
NP → Prn N
S → NP VP
VP → V S
CKY for PCFG parsing

S → NP VP

P(S_{03} \Rightarrow NP_{01} VP_{23}) = P(NP_{01})P(VP_{13})P(S \rightarrow NP VP)
**CKY for PCFG parsing**

\[
\begin{array}{c|c|c|c|c|c}
0 & 1 & 2 & 3 & 4 \\
\hline
\text{Prn, NP} & \text{V, VP} & \text{Prn, NP} & \text{N, V, VP} \\
\text{I} & \text{saw} & \text{her} & \text{duck} \\
\end{array}
\]
CKY for PCFG parsing

I saw her duck

Prn, NP V, VP Prn, NP N, V, VP

S VP NP, S

S → NP VP

VP → V NP

NP → Prn N

S → NP VP

S → NP VP

VP → V S
CKY for PCFG parsing

\[
P(\text{VP}_{14} \Rightarrow V_{12}\text{NP}_{24}) = P(V_{12})P(\text{NP}_{24})P(\text{VP} \rightarrow V \text{ NP})
\]
CKY for PCFG parsing

I saw her duck

S

Prn, NP

V, VP

Prn, NP

N, V, VP

S → NP VP

VP → V NP

NP → Prn N

S → NP VP

S → NP VP

VP → V S

S → NP VP

S → NP VP

S → VP

VP → NP, S

S

V, VP

Prn, NP

N, V, VP

I

saw

her

duck
CKY for PCFG parsing

I saw her duck

S → NP VP
VP → V NP
NP → Prn N

S → NP VP
VP → V S
S → NP VP
CKY for PCFG parsing

\[ S \rightarrow NP \ VP \]

\[
P(S_{14} \Rightarrow NP_{01} VP_{14}) = P(NP_{01}) P(VP_{14}) P(S \rightarrow NP \ VP)
\]
CKY for PCFG parsing

I saw her duck

Prn, NP V, VP Prn, NP N, V, VP

S S V, VP Prn, NP N, V, VP

VP NP, S

S → NP VP

VP → V NP

NP → Prn N

S → NP VP

S → NP VP

VP → V S

S → NP VP
CKY for PCFG parsing
What makes the difference in PCFG probabilities?

<table>
<thead>
<tr>
<th>Rule</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S \Rightarrow NP \ VP$</td>
<td>1.0</td>
</tr>
<tr>
<td>$NP \Rightarrow We$</td>
<td>0.1</td>
</tr>
<tr>
<td>$VP \Rightarrow VP \ PP$</td>
<td>0.1</td>
</tr>
<tr>
<td>$VP \Rightarrow V \ NP$</td>
<td>0.8</td>
</tr>
<tr>
<td>$V \Rightarrow saw$</td>
<td>1.0</td>
</tr>
<tr>
<td>$NP \Rightarrow D \ N$</td>
<td>0.7</td>
</tr>
<tr>
<td>$D \Rightarrow the$</td>
<td>0.4</td>
</tr>
<tr>
<td>$N \Rightarrow man$</td>
<td>0.8</td>
</tr>
<tr>
<td>$PP \Rightarrow P \ NP$</td>
<td>1.0</td>
</tr>
<tr>
<td>$P \Rightarrow with$</td>
<td>1.0</td>
</tr>
<tr>
<td>$NP \Rightarrow D \ N$</td>
<td>0.7</td>
</tr>
<tr>
<td>$D \Rightarrow a$</td>
<td>0.6</td>
</tr>
<tr>
<td>$N \Rightarrow hat$</td>
<td>0.2</td>
</tr>
</tbody>
</table>
What makes the difference in PCFG probabilities?

```
S  ⇒  NP VP  1.0
NP ⇒  We  0.1
VP ⇒  VP PP 0.1
VP ⇒  V NP  0.8
V  ⇒  saw  1.0
NP ⇒  D N  0.7
D ⇒  the  0.4
N ⇒  man  0.8
PP ⇒  P NP  1.0
P ⇒  with  1.0
NP ⇒  D N  0.7
D ⇒  a    0.6
N ⇒  hat  0.2
```

The parser’s choice would not be affected by lexical items!
What is wrong with PCFGs?

- In general: the assumption of independence
- The parents affect the correct choice for children, for example, in English
  \[ NP \rightarrow \text{Prn} \] is more likely in the subject position
- The lexical units affect the correct decision, for example:
  - We eat the pizza with hands
  - We eat the pizza with mushrooms
- Additionally: PCFGs use local context, difficult to incorporate arbitrary/global features for disambiguation
Solutions to PCFG problems

- Independence assumptions can be relaxed by either
  - Parent annotation
  - Lexicalization
- To condition on arbitrary/global information: discriminative models
- Most practical PCFG parsers are lexicalized, and often use a re-ranker conditioning on other (global) features
Lexicalizing PCFGs

- Replace non-terminal $X$ with $X(h)$, where $h$ is a tuple with the lexical word and its POS tag
- Now the grammar can capture (head-driven) lexical dependencies
- But number of nonterminals grow by $|V| \times |T|$
- Estimation becomes difficult (many rules, data sparsity)
- Some treebanks (e.g., Penn Treebank) do not annotate heads, they are automatically annotated (based on heuristics)
Example lexicalized derivation

```
TOP
    \[\rightarrow\] S(bought,VBD)
    \[\rightarrow\] NP(week,NN) \(\rightarrow\) JJ(last,JJ) Last
    \[\rightarrow\] NN(week,NN) week
    \[\rightarrow\] NNP(IBM,NNP) IBM
    \[\rightarrow\] VBD(bought,VBD) bought
    \[\rightarrow\] NP(Lotus,NNP) Lotus

Example rules:
TOP \(\rightarrow\) S(bought,VBD)
S(bought,VBD) \(\rightarrow\) NP(week,NN) NP(IBM,NNP) VP(bought,VBD)
VP(bought,VBD) \(\rightarrow\) VBD(bought,VBD) NP(Lotus,NNP)
JJ(last,JJ) \(\rightarrow\) Last
```
Evaluating the parser output

- A parser can be evaluated
  extrinsically based on its effect on a task (e.g., machine translation) where it is used
  intrinsically based on the match with ideal parsing
- The typically evaluation (intrinsic) is based on a gold standard (GS)
- Exact match is often
  - very difficult to achieve (think about a 50-word newspaper sentence)
  - not strictly necessary (recovering parts of the parse can be useful for many purposes)
Parser evaluation metrics

- Common evaluation metrics are (PARSEVAL):
  - **precision** the ratio of correctly predicted nodes
  - **recall** the nodes (in GS) that are predicted correctly
  - **f-measure** harmonic mean of precision and recall
    \[ \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \]

- The measures can be:
  - *unlabeled* the spans of the nodes are expected to match
  - *labeled* the node label should also match

- Crossing brackets (or average non-crossing brackets)
  - ( We ( saw ( them ( with binoculars )))
  - ( We ( ( saw them ) ( with binoculars )))

- Measures can be averaged per constituent (micro average), or over sentences (macro average)
**PARSEVAL example**

**Gold standard:**

```
S
NP  VP
N    V

NP  NP  PP
D    N    P

We  saw  the  man  with  a  hat
```

**Parser output:**

```
S
NP  VP
N    V

NP  NP  PP
D    N    P

We  saw  the  man  with  a  hat
```

\[
\text{precision} = \frac{6}{7} \quad \text{recall} = \frac{6}{7} \quad \text{f-measure} = \frac{6}{7}
\]
Problems with PARSEVAL metrics

- PARSEVAL metrics favor certain type of structures
  - Results are surprisingly well for flat tree structures (e.g., Penn treebank)
  - Results of some mistakes are catastrophic (e.g., low attachment)
- Not all mistakes are equally important for semantic distinctions
- Some alternatives:
  - Extrinsic evaluation
  - Evaluation based on extracted dependencies
Dependency grammars

- Dependency grammars gained popularity in (particularly in computational) linguistics rather recently, but their roots can be traced back to a few thousand years.
- The main idea is capturing the relation between the words, rather than grouping them into (abstract) constituents.

Note: like constituency grammars, we will not focus on a particular dependency formalism, but discuss it in general in relation to parsing.
Dependency grammars

- No constituents, units of syntactic structure are words

John saw Marry
Dependency grammars

- No constituents, units of syntactic structure are words
- The structure of the sentence is represented by asymmetric binary relations between syntactic units
Dependency grammars

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- The links (relations) have labels (dependency types)
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Dependency grammars

- No constituents, units of syntactic structure are words
- The structure of the sentence is represented by asymmetric binary relations between syntactic units
- The links (relations) have labels (dependency types)
- Each relation defines one of the words as the head and the other as dependent
- Often an artificial root node is used for computational convenience
Projective vs. non-projective dependencies

- If a dependency graph has no crossing edges, it is said to be projective, otherwise non-projective.
- Non-projectivity stems from long-distance dependencies and free word order.
Projective vs. non-projective dependencies

- If a dependency graph has no crossing edges, it is said to be *projective*, otherwise *non-projective*
- Non-projectivity stems from long-distance dependencies and free word order

A non-projective tree example:
Parsing with dependency grammars

- Projective parsing can be done in polynomial time
- Non-projective parsing is NP-hard (without restrictions)
- For both, it is a common practice to use greedy (e.g., linear time) algorithms
A dependency grammar is a tuple \((V, A)\)

- \(V\) is a set of nodes corresponding to the (syntactic) words (we implicitly assume that words have indexes)
- \(A\) is a set of arcs of the form \((w_i, r, w_j)\) where
  - \(w_i \in V\) is the head
  - \(r\) is the type of the relation (arc label)
  - \(w_j \in V\) is the dependent

This defines a directed graph.
Dependency grammars: common assumptions

- Every word has a single head
- The dependency graphs are acyclic
- The graph is connected
- With these assumptions, the representation is a tree
- Note that these assumptions are not universal but common for dependency parsing
Dependency parsing

- Dependency parsing has many similarities with context-free parsing (e.g., trees)
- They also have some different properties (e.g., number of edges and depth of trees are limited)
- Dependency parsing can be
  - grammar-driven (hand crafted rules or constraints)
  - data-driven (rules/model is learned from a treebank)
- There are two main approaches:
  Graph-based similar to context-free parsing, search for the best tree structure
  Transition-based similar to shift-reduce parsing (used for programming language parsing), but using greedy search for the best transition sequence
Transition based parsing

- Inspired by shift-reduce parsing, single pass over the input
- Use a stack and a buffer of unprocessed words
- Parsing as predicting a sequence of transitions like
  - **Left-Arc**: mark current word as the head of the word on top of the stack
  - **Right-Arc**: mark current word as a dependent of the word on top of the stack
  - **Shift**: push the current word to the stack
- Algorithm terminates when all words in the input are processed
- The transitions are not naturally deterministic, best transition is predicted using a machine learning method

(Yamada and Matsumoto 2003; Nivre, Hall, and Nilsson 2004)
A typical transition system

\[
(σ | \begin{array}{c} w_i \end{array}, \begin{array}{c} stack top \end{array}, \begin{array}{c} stack \end{array}, \begin{array}{c} next word \end{array}, \begin{array}{c} w_j \end{array}, \begin{array}{c} buffer \end{array}, \begin{array}{c} \beta \end{array}, \begin{array}{c} A \end{array}, \begin{array}{c} arcs \end{array})
\]

**Left-Arc**: \((σ|w_i, w_j|β, A) \Rightarrow (σ, w_j|β, A \cup \{(w_j, r, w_i)\})\)
- pop \(w_i\)
- add arc \((w_j, r, w_i)\) to \(A\)
- keep \(w_j\) in the buffer

**Right-Arc**: \((σ|w_i, w_j|β, A) \Rightarrow (σ, w_i|β, A \cup \{(w_i, r, w_j)\})\)
- pop \(w_i\)
- add arc \((w_i, r, w_j)\) to \(A\)
- move \(w_i\) to the buffer (\(w_j\) is removed from the buffer)

**Shift**: \((σ, w_j|β, A) \Rightarrow (σ|w_j, β, A)\)
- push \(w_j\) to the stack (remove it from the buffer)

(Kübler, McDonald, and Nivre 2009, p.23)
Transition based parsing: example
Transition based parsing: example

We saw her with binoculars

Note: we need Shift for NP attachment.
Transition based parsing: example

We saw her with binoculars

Note: we need **Shift** for NP attachment.
Transition based parsing: example

Note: We need SHIFT for NP attachment.
Transition based parsing: example

We saw her with binoculars

Note: we need Shift for NP attachment.
Transition based parsing: example

We saw her with binoculars

Note: we need **Shift** for NP attachment.

Root \(\rightarrow\) We \(\rightarrow\) saw \(\rightarrow\) her

nsubj \(\rightarrow\) obj

buffer \(\rightarrow\) with \(\rightarrow\) binoculars

**Shift**
Transition based parsing: example

We saw her with binoculars

Note: we need Shift for NP attachment.

Note: We need Shift for NP attachment.
Transition based parsing: example

We saw her with binoculars

Note: we need \textit{Shift} for NP attachment.

Note: We need \textit{Shift} for NP attachment.
Transition based parsing: example

We saw her with binoculars

Note: we need **Shift** for NP attachment.

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Transition based parsing: example

We saw her with binoculars.

Note: we need Shift for NP attachment.

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Transition based parsing: example

We saw her with binoculars

Note: we need **Shift** for NP attachment.
Making transition decisions

- In shift-reduce parsing the actions are deterministic
- In transition-based dependency parsing, we need to choose among all possible transitions
- The typical method is to train a (discriminative) classifier on features extracted from gold-standard transition sequences
- Almost any machine learning method is applicable. Common choices include
  - Memory-based learning
  - Support vector machines
  - (Deep) neural networks
Features for transition-based parsing

• The features come from certain ‘addresses’ in the parser configuration, for example
  – The word at the stack top (or nth from stack top)
  – The first/second word on the buffer
  – Right/left dependents of the word on top of the stack/buffer

• For each possible ‘address’, we can make use of features like
  – Word form, lemma, POS tag, morphological features, word embeddings
  – Dependency relations – \((w_i, r, w_j)\) triples

• Note that for some ‘address’–‘feature’ combinations and in some configurations the values may be missing
The training data

- We want features like,
  - lemma[Stack] = duck
  - POS[Stack] = NOUN
    - ...

- But treebank gives us:

```
1  Read  read  VERB  VB  Mood=Imp|VerbForm=Fin  0  root
2   on   on    ADV  RB   _       1  advmod
3   to   to    PART  TO   _       4  mark
4  learn learn  VERB  VB  VerbForm=Inf  1  xcomp
5    the  the  DET   DT  Definite=Def  6  det
6  facts  fact  NOUN  NNS  Number=Plur  4  obj
7  .     .     PUNCT .   _       1  punct
```

- The treebank has the outcome of the parser, but not the features we need
The training data

- The features for transition-based parsing have to be from *parser configurations*
- The data (treebanks) need to be preprocessed for obtaining the training data
- Construct a transition sequence by parsing the sentences, and using treebank annotations (the set $A$) as an ‘oracle’
- Decide for
  - $\text{LEFT-ARC}_T$ if $(\beta[0], r, \sigma[0]) \in A$
  - $\text{RIGHT-ARC}_T$ if $(\sigma[0], r, \beta[0]) \in A$
  - $\text{SHIFT}$ otherwise
- There may be multiple sequences that yield the same dependency tree, the above defines a ‘canonical’ transition sequence
Non-projective parsing

- The transition-based parsing we defined so far works only for projective dependencies.
- One way to achieve (limited) non-projective parsing is to add special **Left-Arc** and **Right-Arc** transitions to/from non-top words from the stack.
- Another method is pseudo-projective parsing:
  - preprocessing to ‘projectivize’ the trees before training
    - The idea is to attach the dependents to a higher level head that preserves projectivity, while marking it on the new dependency label
  - postprocessing for restoring the projectivity after parsing
    - Re-introduce projectivity for the marked dependencies
Pseudo-projective parsing

Non-projective tree:

Pseudo-projective tree:
Transition based parsing: summary/notes

- Linear time, greedy parsing
- Can be extended to non-projective dependencies
- One can use arbitrary features
- We need some extra work for generating gold-standard transition sequences from treebanks
- Early errors propagate, transition-based parsers make more mistakes on long-distance dependencies
- The greedy algorithm can be extended to beam search for better accuracy (still linear time complexity)
Graph-based parsing: preliminaries

- Enumerate all possible dependency trees
- Pick the best scoring tree
- Features are based on limited parse history (like CFG parsing)
- Two well-known flavors:
  - Maximum (weight) spanning tree (MST)
  - Chart-parsing based methods

Eisner 1996; McDonald et al. 2005
MST parsing: preliminaries

Spanning tree of a graph

- Spanning tree of a connected graph is a sub-graph which is a tree and traverses all the nodes.
MST parsing: preliminaries

Spanning tree of a graph

- Spanning tree of a connected graph is a sub-graph which is a tree and traverses all the nodes
- For fully-connected graphs, the number of spanning trees are exponential in the size of the graph
- The problem is well studied
- There are efficient algorithms for enumerating and finding the optimum spanning tree on weighted graphs
MST algorithm for dependency parsing

- For directed graphs, there is a polynomial time algorithm that finds the minimum/maximum spanning tree (MST) of a fully connected graph (Chu-Liu-Edmonds algorithm)
- The algorithm starts with a dense/fully connected graph
- Removes edges until the resulting graph is a tree
MST example

For each node select the incoming arc with highest weight.
MST example

Detect cycles, contract them to a ‘single node’
MST example

Pick the best arc into the combined node, break the cycle
MST example

Once all cycles are eliminated, the result is the MST.
Properties of the MST parser

- The MST parser is non-projective
- There is an algorithm with $O(n^2)$ time complexity (Tarjan 1977)
- The time complexity increases with typed dependencies (but still polynomial)
- The weights/parameters are associated with edges (often called ‘arc-factored’)  

- We can learn the arc weights directly from a treebank
- However, it is difficult to incorporate non-local features
CKY for dependency parsing

- The CKY algorithm can be adapted to projective dependency parsing
- For a naive implementation the complexity increases drastically $O(n^6)$
  - Any of the words within the span can be the head
  - Inner loop has to consider all possible splits
- For projective parsing, the observation that the left and right dependents of a head are independently generated reduces the complexity to $O(n^3)$

(Eisner 1997)
Non-local features

- The graph-based dependency parsers use edge-based features.
- This limits the use of more global features.
- Some extensions for using ‘more’ global features are possible.
- This often leads non-projective parsing to become intractable.
External features

- For both types of parsers, one can obtain features that are based on unsupervised methods such as
  - clustering
  - dense vector representations (embeddings)
  - alignment/transfer from bilingual corpora/treebanks

(Koo, Carreras, and Collins 2008)
Errors from different parsers

- Different parsers make different errors
  - Transition based parsers do well on local arcs, worse on long-distance arcs
  - Graph based parsers tend to do better on long-distance dependencies

- Parser combination is a good way to combine the powers of different models.
  Two common methods
  - Majority voting: train parsers separately, use the weighted combination of their results
  - Stacking: use the output of a parser as features for another

(McDonald and Satta 2007; Sagae and Lavie 2006; Nivre and McDonald 2008)
Evaluation metrics for dependency parsers

- Like CF parsing, exact match is often too strict
- *Attachment score* is the ratio of words whose heads are identified correctly.
  - *Labeled attachment score* (LAS) requires the dependency type to match
  - *Unlabeled attachment score* (UAS) disregards the dependency type
- *Precision/recall/F-measure* often used for quantifying success on identifying a particular dependency type
  - **precision** is the ratio of correctly identified dependencies (of a certain type)
  - **recall** is the ratio of dependencies in the gold standard that parser predicted correctly
  - **f-measure** is the harmonic mean of precision and recall \( \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \)
Evaluation example

**Gold standard**

```
I saw her duck
```

**Parser output**

```
I saw her duck
```

UAS 100%
LAS 50%
Precision_{nsubj} 50%
Recall_{nsubj} 100%
Precision_{obj} 0%
Recall_{obj} 0%
Evaluation example

Gold standard

Parser output

I saw her duck

UAS 100%
LAS
Precision$_{nsubj}$
Recall$_{nsubj}$
Precision$_{obj}$
Recall$_{obj}$
Evaluation example

Gold standard

```
I saw her duck
```

```
root
```

```
nsubj
```

```
obj
```

```
nmod
```

Parser output

```
I saw her duck
```

```
root
```

```
nsubj
```

```
ccomp
```

```
nsubj
```

UAS 100%
LAS 50%

Precision\textsubscript{nsubj} 50%
Recall\textsubscript{nsubj} 100%
Precision\textsubscript{obj} 0%
Recall\textsubscript{obj} 0%
Evaluation example

**Gold standard**

```
root

nsubj
I

obj
saw

nmod
her

duck
```

**Parser output**

```
root

nsubj
I

ccomp
saw

nsubj
her

nsubj
duck
```

<table>
<thead>
<tr>
<th>Measure</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAS</td>
<td>100%</td>
</tr>
<tr>
<td>LAS</td>
<td>50%</td>
</tr>
<tr>
<td>Precision$_{nsubj}$</td>
<td>50%</td>
</tr>
<tr>
<td>Recall$_{nsubj}$</td>
<td></td>
</tr>
<tr>
<td>Precision$_{obj}$</td>
<td></td>
</tr>
<tr>
<td>Recall$_{obj}$</td>
<td></td>
</tr>
</tbody>
</table>
Evaluation example

Gold standard

Parser output

UAS 100%
LAS 50%
Precision_{nsubj} 50%
Recall_{nsubj} 100%
Precision_{obj} (assumed)
Recall_{obj} 0%
Evaluation example

Gold standard

Parser output

UAS 100%
LAS 50%
Precision_{nsubj} 50%
Recall_{nsubj} 100%
Precision_{obj} 0% (assumed)
Recall_{obj}
Evaluation example

**Gold standard**

```
I saw her duck
```

**Parser output**

```
I saw her duck
```

<table>
<thead>
<tr>
<th>Measure</th>
<th>UAS</th>
<th>LAS</th>
<th>Precision (<em>n</em>{subj})</th>
<th>Precision (_obj)</th>
<th>Recall (<em>n</em>{subj})</th>
<th>Recall (_obj)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>100%</td>
<td>50%</td>
<td>50%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>
Averaging evaluation scores

• As in context-free parsing, average scores can be
  macro-average or sentence-based
  micro-average or word-based

• Consider a two-sentence test set with
  
<table>
<thead>
<tr>
<th>words</th>
<th>correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>sentence 1</td>
<td>30</td>
</tr>
<tr>
<td>sentence 2</td>
<td>10</td>
</tr>
</tbody>
</table>

  – word-based average attachment score:
  – sentence-based average attachment score:
Averaging evaluation scores

- As in context-free parsing, average scores can be macro-average or sentence-based micro-average or word-based

- Consider a two-sentence test set with

<table>
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<tr>
<td>sentence 2</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

- word-based average attachment score: 50% (20/40)
- sentence-based average attachment score: 66% ((1 + 1/3)/2)
Dependency parsing: summary

- Dependency relations are often easier to interpret
- It is also claimed that dependency parsers are more suitable for parsing free-word-order languages
- Dependency relations are between words, no phrases or other abstract nodes are postulated
- Two general methods:
  - transition based: greedy search, non-local features, fast, less accurate
  - graph based: exact search, local features, slower, accurate (within model limitations)
- Combination of different methods often result in better performance
- Non-projective parsing is more difficult
- Most of the recent parsing research has focused on better machine learning methods (mainly using neural networks)
Next week

Mon/Wed  Wrap-up/summary
Fri      Exam
Where to go from here?

- Textbook includes good coverage of constituency grammars and parsing, online 3rd edition includes a chapter on dependency parsing as well.
- The book by Kübler, McDonald, and Nivre (2009) is an accessible introduction to (statistical) dependency parsing.
- For more on linguistic and mathematical foundations of parsing:
  - Müller (2016) is a new open-source text book on Grammar formalisms.
  - Aho and Ullman (1972) is the classical reference (available online) for parsing (programming languages) and also includes discussion of grammar classes in the Chomsky hierarchy. A more up-to-date alternative is Aho, Lam, et al. (2007).
  - There is a brief introductory section on dependency grammars in Kübler, McDonald, and Nivre (2009), for a classical reference see tesniere2015, English translation of the original version (Tesnière 1959).
Pointers to some treebanks

Treebanks are the main resource for statistical parsing. A few treebank-related resources to have a look at until next time:

- Universal dependencies project, documentation, treebanks:
  http://universaldependencies.org/

- Tübingen treebanks:
  TüBa-D/Z written German
  TüBa-D/S spoken German
  TüBa-E/S spoken English
  TüBa-J/S spoken Japanese

- TüNDRA - a treebank search and visualization application with the above treebanks and few more
  - Main version:
    https://weblicht.sfs.uni-tuebingen.de/Tundra/
  - New version (beta):
    https://weblicht.sfs.uni-tuebingen.de/tundra-beta/
CKY algorithm

function CKY(words, grammar)
    for j ← 1 to LENGTH(words) do
        table[j − 1, j] ← {A|A → words[j] ∈ grammar}
    for i ← j − 1 downto 0 do
        for k ← i + 1 to j − 1 do
            table[i, j] ← table[i, j] ∪
            {A|A → BC ∈ grammar and
                B ∈ table[i, k] and
                C ∈ table[k, j]}
    return table
Even more examples
(newspaper headlines)

- FARMER BILL DIES IN HOUSE
- TEACHER STRIKES IDLE KIDS
- SQUAD HELPS DOG BITE VICTIM
- BAN ON NUDE DANCING ON GOVERNOR’S DESK
- PROSTITUTES APPEAL TO POPE
- KIDS MAKE NUTRITIOUS SNACKS
- DRUNK GETS NINE MONTHS IN VIOLIN CASE
- MINERS REFUSE TO WORK AFTER DEATH
Another CKY demonstration: spans

she saw a duck

S → NP VP
S → Aux X
X → NP VP
NP → Det N
NP → she | her
NP → NP PP
VP → V NP
VP → duck|saw|...
VP → VP PP
PP → Prp NP
N → duck
N → park
N → parks
V → duck
V → ducks
V → saw
Prn → she | her
Prp → in | with
Det → a | the
Another CKY demonstration: spans

0. she
1. saw
2. a
3. duck

S → NP VP
S → Aux X
X → NP VP
NP → Det N
NP → she | her
NP → NP PP
VP → V NP
VP → duck|saw|...
VP → VP PP
PP → Prp NP
N → duck
N → park
N → parks
V → duck
V → ducks
V → saw
Prn → she | her
Prp → in | with
Det → a | the
Another CKY demonstration: spans

\[
S \rightarrow NP \ VP \\
S \rightarrow \text{Aux} \ X \\
X \rightarrow NP \ VP \\
NP \rightarrow \text{Det} \ N \\
NP \rightarrow \text{she} \ | \ \text{her} \\
NP \rightarrow NP \ PP \\
VP \rightarrow V \ NP \\
VP \rightarrow \text{duck|saw|...} \\
VP \rightarrow VP \ PP \\
PP \rightarrow \text{Prp} \ NP \\
N \rightarrow \text{duck} \\
N \rightarrow \text{park} \\
N \rightarrow \text{parks} \\
V \rightarrow \text{duck} \\
V \rightarrow \text{ducks} \\
V \rightarrow \text{saw} \\
\text{Prn} \rightarrow \text{she} \ | \ \text{her} \\
\text{Prp} \rightarrow \text{in} \ | \ \text{with} \\
\text{Det} \rightarrow \text{a} \ | \ \text{the}
\]

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Summer Semester 2020
Another CKY demonstration: spans

S → NP VP
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Another CKY demonstration: spans

\[
\begin{align*}
S & \rightarrow NP \ VP \\
S & \rightarrow Aux \ X \\
X & \rightarrow NP \ VP \\
NP & \rightarrow Det \ N \\
NP & \rightarrow she \ | \ her \\
NP & \rightarrow NP \ PP \\
VP & \rightarrow V \ NP \\
VP & \rightarrow duck|saw|... \\
VP & \rightarrow VP \ PP \\
PP & \rightarrow Prp \ NP \\
N & \rightarrow duck \\
N & \rightarrow park \\
N & \rightarrow parks \\
V & \rightarrow duck \\
V & \rightarrow ducks \\
V & \rightarrow saw \\
Prn & \rightarrow she \ | \ her \\
Prp & \rightarrow in \ | \ with \\
Det & \rightarrow a \ | \ the
\end{align*}
\]